

The effect of convection on pulsational stability

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Abstract

A review on the current state of mode physics in classical pulsators is presented. Two, currently in use, time-dependent convection models are compared and their applications on mode stability are discussed with particular emphasis on the location of the Delta Scuti instability strip.

Introduction

Stars with relatively low surface temperatures show distinctive envelope convection zones which affect mode stability. Among the first problems of this nature was the modelling of the red edge of the classical instability strip (IS) in the Hertzsprung-Russell (H-R) diagram. The first pulsation calculations of classical pulsators without any pulsation-convection modelling predicted red edges which were much too cool and which were at best only neutrally stable. What follows were several attempts to bring the theoretically predicted location of the red edge in better agreement with the observed location by using time-dependent convection models in the pulsation analyses (Dupree 1977; Baker & Gough 1979; Gonzi 1982; Stellingwerf 1984). More recently several authors, e.g. Bono et al. (1995, 1999), Houdek (1997, 2000), Xiong & Deng (2001, 2007), Dupret et al. (2005) were successful to model the red edge of the classical IS. These authors report, however, that different physical mechanisms are responsible for the return to stability. For example, Bono et al. (1995) and

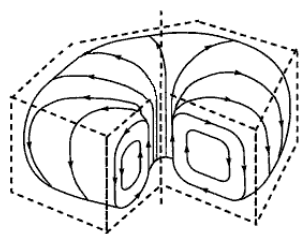


Figure 1: Sketch of an overturning hexagonal (dashed lines) convective cell. Near the centre the gas raises from the hot bottom to the cooler top (surface) where it moves nearly horizontally towards the edges, thereby losing heat. The cooled gas then descends along the edges to close the circular flow. Arrows indicate the direction of the flow pattern.

Table 1: Summary of time-dependent convection model differences.

Balance between buoyancy & turbulent drag (Unno 1967, 1977)	Kinetic theory of accelerating eddies (Gough 1965, 1977a)
- acceleration terms of convective fluctuations w, T' neglected	- acceleration terms included: w, T' evolve with growth rate σ
- nonlinear terms approximated by spatial gradients $\propto 1/\ell$	- nonlinear terms are neglected during eddy growth
- $\partial_i p'$ neglected in momentum equ.	- $\partial_i p'$ included in Eq. (1)
- characteristic eddy lifetime: $\tau \simeq \ell/2w$	- $\tau = 2/\sigma$ determined stochastically from parametrized shear instability
- variation $\ell_1 = \delta\ell/\ell$ (Unno 1967): $\omega\tau < 1$: $\ell_1 \sim H_1$ $\omega\tau > 1$: $\ell_1 \sim r_1$ or (Unno 1977): $\ell_1 \sim (1 + i\omega^2\tau^2)^{-1}(H_1 - i\omega\tau\rho_1/3)$ (H is pressure scale height)	- variation of mixing length according to rapid distortion theory (Townsend 1976), i.e. variation also of eddy shape
- turbulent pressure p_t neglected in hydrostatic support equation	- $p_t = \overline{\rho w w}$ included in mean equ. for hydrostatic support

Dupret et al. (2005) report that it is mainly the convective heat flux, Xiong & Deng (2001) the turbulent viscosity, and Baker & Gough (1979) and Houdek (2000) predominantly the momentum flux (turbulent pressure p_t) that stabilizes the pulsation modes at the red edge.

Time-dependent convection models

The authors mentioned in the previous section used different implementations for modelling the interaction of the turbulent velocity field with the pulsation. In the past various time-dependent convection models were proposed, for example, by Schatzman (1956), Gough (1965, 1977a), Unno (1967, 1977), Xiong (1977, 1989), Stellingwerf (1982), Kuhfuß (1986), Canuto (1992), Gabriel (1996), Grigahcène et al. (2005). Here I shall briefly review and compare the basic concepts of two, currently in use, convection models. The first model is that by Gough (1977a,b), which has been used, for example, by Baker & Gough (1979), Balmforth (1992) and by Houdek (2000). The second model is that by Unno (1967, 1977), upon which the generalized models by Gabriel (1996) and Grigahcène et al. (2005) are based, with applications by Dupret et al. (2005).

Nearly all of the time-dependent convection models assume the Boussinesq approximation to the equations of motion. The Boussinesq approximation relies on the fact that the height of the fluid layer is small compared with the density scale height. It is based on a careful scaling argument and an expansion in small parameters (Spiegel & Veronis 1960; Gough 1969). The fluctuating convection equations for an inviscid Boussinesq fluid in a static plane-parallel atmosphere are

$$\partial_t u_i + (u_j \partial_j u_i - \overline{u_j \partial_j u_i}) = -\bar{\rho}^{-1} \partial_i p' + g \hat{\alpha} T' \delta_{i3}, \quad (1)$$

$$\partial_t T' + (u_j \partial_j T' - \overline{u_j \partial_j T'}) = \beta w - (\bar{\rho} c_p)^{-1} \partial_i F'_i, \quad (2)$$

supplemented by the continuity equation for an incompressible gas, $\partial_j u_j = 0$, where $\mathbf{u} = (u, v, w)$ is the turbulent velocity field, ρ is density, p is gas pressure, g is the acceleration due to gravity, T is temperature, c_p is the specific heat at constant pressure, $\hat{\alpha} = -(\partial \ln \rho / \partial \ln T)_p / T$, F_i is the radiative heat flux, β is the superadiabatic temperature gradient and δ_{ij} is the Kronecker delta. Primes (') indicate Eulerian fluctuations and overbars horizontal averages. These are the starting equations for the two physical pictures describing the motion of an overturning convective eddy, illustrated in Fig. 1.

In the first physical picture, adopted by Unno (1967), the turbulent element, with a characteristic vertical length ℓ , evolves out of some chaotic state and achieves steady motion very quickly. The fluid element maintains exact balance between buoyancy force and turbulent drag by continuous exchange of momentum with other elements and its surroundings. Thus the acceleration terms $\partial_t u_i$ and $\partial_t T'$ are neglected and the nonlinear advection terms provide dissipation (of kinetic energy) that balances the driving terms. The nonlinear advection terms are approximated by $u_j \partial_j u_i - \overline{u_j \partial_j u_i} \simeq 2w^2/\ell$ and $u_j \partial_j T' - \overline{u_j \partial_j T'} \simeq 2wT'/\ell$. This leads to two nonlinear equations which need to be solved numerically together with the mean equations of the stellar structure.

The second physical picture, which was generalized by Gough (1965, 1977a,b) to the time-dependent case, interprets the turbulent flow by indirect analogy with kinetic gas theory. The motion is not steady and one imagines the convective element to accelerate from rest followed by an instantaneous breakup after the element's lifetime. Thus the nonlinear advection terms are neglected in the convective fluctuation equations (1)-(2) but are taken to be responsible for the creation and destruction of the convective eddies (Gough 1977a,b). By retaining only the acceleration terms the equations become linear with analytical solutions $w \propto \exp(\sigma t)$ and $T' \propto \exp(\sigma t)$ subject to proper periodic spatial boundary conditions, where t is time and $\Re(\sigma)$ is the linear convective growth rate. The mixing length ℓ enters in the calculation of the eddy's survival probability, which is proportional to the eddy's internal shear (rms vorticity), for determining the convective heat and momentum fluxes. Although

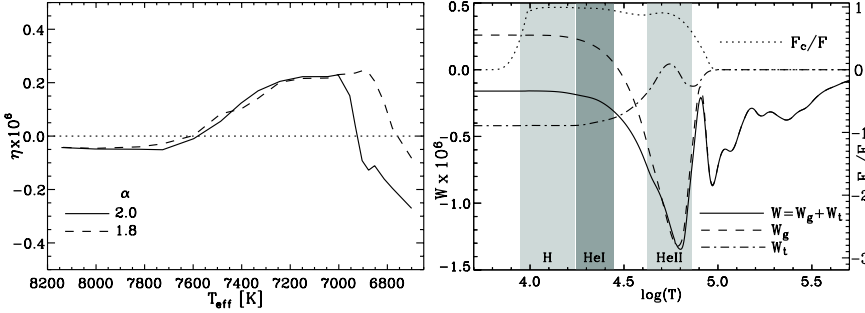


Figure 2: Mode stability of an $1.7 M_{\odot}$ Delta Scuti star computed with Gough's (1977a,b) convection model. Left: Stability coefficient $\eta = \omega_i/\omega_r$ as a function of surface temperature T_{eff} across the IS. Results are shown for the fundamental radial mode ($n = 1$) and for two values of the mixing-length parameter α . Positive η values indicate mode instability. Right: Integrated work integral W as a function of the depth co-ordinate $\log(T)$ for a model lying just outside the cool edge of the IS ($T_{\text{eff}} = 6813$ K). Results are plotted in units of η and for $\alpha = 2.0$. Contributions to W (solid curve) arising from the gas pressure perturbation, W_g (dashed curve), and the turbulent pressure fluctuations, W_t (dot-dashed curve), are indicated ($W = W_g + W_t$). The dotted curve is the ratio of the convective to the total heat flux F_c/F . Ionization zones of H and He (5% to 95% ionization) are indicated (from Houdek 2000).

the two physical pictures give the same result in a static envelope, the results for the fluctuating turbulent fluxes in a pulsating star are very different (Gough 1977a). The main differences between Unno's and Gough's convection model are summarized in Table 1.

Application on mode stability in δ Scuti stars

Fig. 2 displays the mode stability of an evolving $1.7 M_{\odot}$ Delta Scuti star crossing the IS. The results were computed with the time-dependent, nonlocal convection model by Gough (1977a,b). As demonstrated in the right panel of Fig. 2, the dominating damping term to the work integral W for a star located near the red edge is the contribution from the turbulent pressure fluctuations W_t .

Gabriel (1996) and more recently Grigahcène et al. (2005) generalized Unno's time-dependent convection model for stability computations of nonradial oscillation modes. They included in their mean thermal energy equation the viscous dissipation of turbulent kinetic energy, ϵ , as an additional heat source. The dissipation of turbulent kinetic energy is introduced in the conservation equation for the turbulent kinetic energy $K := \overline{u_i u_i}/2$ (e.g. Tennekes & Lumley 1972, §3.4; Canuto 1992; Houdek & Gough 1999):

$$D_t K + \partial_j (\overline{K u_j} + \bar{p}^{-1} \overline{p' u_j}) - \nu \partial_i^2 K = -\overline{u_i u_i} \partial_j U_i + g \hat{\alpha} \overline{u_j T'} - \epsilon, \quad (3)$$

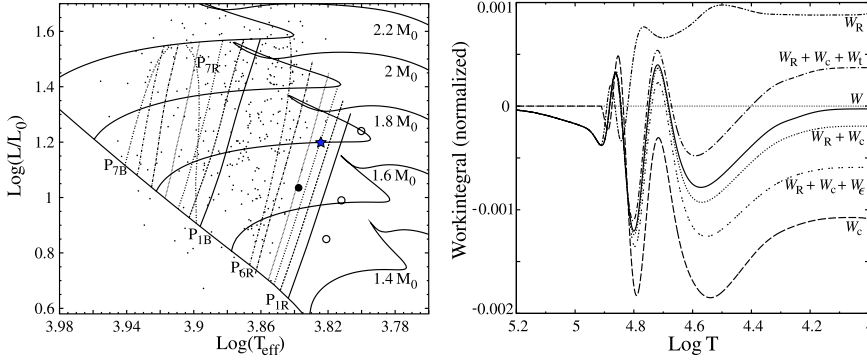


Figure 3: Stability computations of Delta Scuti stars which include the viscous dissipation rate ϵ of turbulent kinetic energy according to Grigahcène et al. (2005). Left: Blue and red edges of the IS superposed on evolutionary tracks on the theorists H-R diagram. The locations of the edges, labelled p_{nB} and p_{nR} , are indicated for radial modes with orders $1 \leq n \leq 7$. Results by Houdek (2000, $\alpha = 2.0$, see Fig. 2) and Xiong & Deng (2001) for the gravest p modes are plotted as the filled and open circles respectively. Right: Integrated work integral W as a function of the depth co-ordinate $\log(T)$ for a stable $n = 3$ radial mode of a $1.8 M_{\odot}$ star (see ‘star’ symbol in the left panel). Contributions to W arising from the radiative flux, W_R , the convective flux, W_c , the turbulent pressure fluctuations, W_t ($W_R + W_c + W_t$), and from the perturbation of the turbulent kinetic energy dissipation, W_{ϵ} ($W_R + W_c + W_{\epsilon}$), are indicated (adapted from Dupret et al. 2005).

where D_t is the material derivative, U_i is the average (oscillation) velocity, i.e. the total velocity $\tilde{u}_i = U_i + u_i$, and ν is the constant kinematic viscosity (in the limit of high Reynolds numbers the molecular transport term can be neglected). The first and second term on the right of Eq. (3) are the shear and buoyant productions of turbulent kinetic energy, whereas the last term $\epsilon = \nu(\partial_j u_i + \partial_i u_j)^2/2$ is the viscous dissipation of turbulent kinetic energy into heat. This term is also present in the mean thermal energy equation, but with opposite sign. The linearized perturbed mean thermal energy equation for a star pulsating radially with complex angular frequency $\omega = \omega_r + i\omega_i$ can then be written, in the absence of nuclear reactions, as (‘ δ ’ denotes a Lagrangian fluctuation and I omit overbars in the mean quantities):

$$d\delta L/dm = -i\omega c_p T(\delta T/T - \nabla_{ad}\delta p/p) + \delta\epsilon, \quad (4)$$

where m is the radial mass co-ordinate, $\nabla_{ad} = (\partial \ln T / \partial \ln p)_s$ and L is the total (radiative and convective) luminosity. Grigahcène et al. (2005) evaluated ϵ from a turbulent kinetic energy equation which was derived without the assumption of the Boussinesq approximation. Furthermore it is not obvious whether

the dominant buoyancy production term, $g\widehat{\alpha}u_j\overline{T'}$ (see Eq. 3), was included in their turbulent kinetic energy equation and so in their expression for ϵ .

Dupret et al. (2005) applied the convection model of Grigahcène et al. (2005) to Delta Scuti and γ Doradus stars and reported well defined red edges. The results of their stability analysis for Delta Scuti stars are depicted in Fig. 3. The left panel compares the location of the red edge with results reported by Houdek (2000, see also Fig. 2) and Xiong & Deng (2001). The right panel of Fig. 3 displays the individual contributions to the accumulated work integral W for a star located near the red edge of the $n = 3$ mode (indicated by the ‘star’ symbol in the left panel). It demonstrates the near cancellation effect between the contributions of the turbulent kinetic energy dissipation, W_ϵ , and turbulent pressure, W_t , making the contribution from the fluctuating convective heat flux, W_c , the dominating damping term. The near cancellation effect between W_ϵ and W_t was demonstrated first by Ledoux & Walraven (1958, §65) (see also Gabriel 1996) by writing the sum of both work integrals as:

$$W_\epsilon + W_t = 3\pi/2 \int_{m_b}^M (5/3 - \gamma_3) \Im(\delta p_t^* \delta \rho) \rho^{-2} dm, \quad (5)$$

where M is the stellar mass, m_b is the enclosed mass at the bottom of the envelope and $\gamma_3 \equiv 1 + (\partial \ln T / \partial \ln \rho)_s$ (s is specific entropy) is the third adiabatic exponent. Except in ionization zones $\gamma_3 \simeq 5/3$ and consequently $W_\epsilon + W_t \simeq 0$.

The convection model by Xiong (1977, 1989) uses transport equations for the second-order moments of the convective fluctuations. In the transport equation for the turbulent kinetic energy Xiong adopts the approximation by Hinze (1975) for the turbulent dissipation rate, i.e. $\epsilon = 2\chi k(\overline{u_i u_i} \rho^2 / 3\rho^2)^{3/2}$, where $\chi = 0.45$ is the Heisenberg eddy coupling coefficient and $k \propto \ell^{-1}$ is the wavenumber of the energy-containing eddies. However, Xiong does not provide a work integral for ϵ (neither does Unno et al. 1989, §26,30) but includes the viscous damping effect of the small-scale turbulence in his model. The convection models considered here describe only the largest, most energy-containing eddies and ignore the dynamics of the small-scale eddies lying further down the turbulent cascade. Small-scale turbulence does, however, contribute directly to the turbulent fluxes and, under the assumption that they evolve isotropically, they generate an effective viscosity ν_t which is felt by a particular pulsation mode as an additional damping effect. The turbulent viscosity can be estimated as (e.g. Gough 1977b; Unno et al. 1989, §20) $\nu_t \simeq \lambda(\overline{w w})^{1/2} \ell$, where λ is a parameter of order unity. The associated work integral W_ν can be written in Cartesian co-ordinates as (Ledoux & Walraven 1958, §63)

$$W_\nu = -2\pi \omega_r \int_{m_b}^M \nu_t \left[e_{ij} e_{ij} - \frac{1}{3} (\nabla \cdot \xi)^2 \right] dm, \quad (6)$$

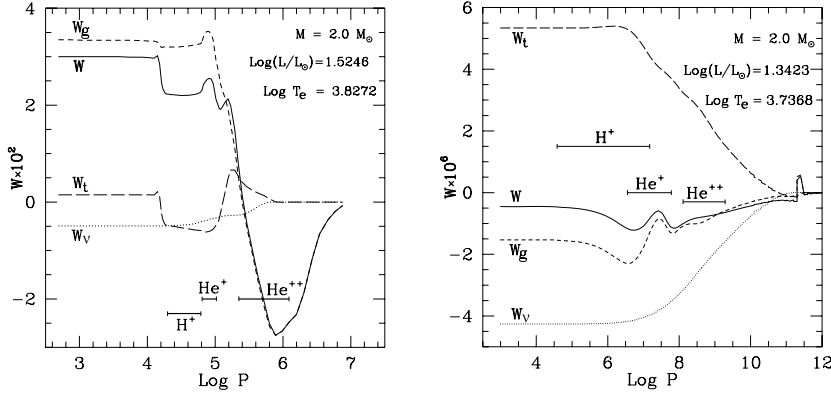


Figure 4: Accumulated work integral W as a function of the depth co-ordinate $\log(p)$. Results are shown for the $n = 1$ radial mode of a Delta Scuti star located inside the IS (left panel) and outside the red edge of the IS (right panel). The stability calculations include viscous dissipation by the small-scale turbulence (Xiong 1989; see Eq. 6). Contributions to W (solid curve) arising from the fluctuating gas pressure, W_g (dashed curve), the turbulent pressure perturbations, W_t (long-dashed curve), and from the turbulent viscosity, W_v (dotted curve), are indicated ($W = W_g + W_t + W_v$). The ionization zones of H and He are marked (adapted from Xiong & Deng 2007).

where $e_{ij} = (\partial_j \xi_i + \partial_i \xi_j)/2$ and ξ is the displacement eigenfunction. Xiong & Deng (2001, 2007) modelled successfully the IS of Delta Scuti and red giant stars and found the dominating damping effect to be the turbulent viscosity (Eq. 6). This is illustrated in Fig. 4 for two Delta Scuti stars: one is located inside the IS (left panel), the other outside the cool edge of the IS (right panel). The contribution from the small-scale turbulence was also the dominant damping effect in the stability calculations by Xiong et al. (2000) of radial p modes in the Sun, although the authors still found unstable modes with orders between $11 \leq n \leq 23$. The importance of the turbulent damping was reported first by Goldreich & Keeley (1977) and later by Goldreich & Kumar (1991), who found all solar modes to be stable only if turbulent damping was included in their stability computations. In contrast, Balmforth (1992), who adopted the convection model of Gough (1977a,b), found all solar p modes to be stable due mainly to the damping of the turbulent pressure perturbations, W_t , and reported that viscous damping, W_v , is about one order of magnitude smaller than the contribution of W_t . Turbulent viscosity (Eq. 6) leads always to mode damping, where as the perturbation of the turbulent kinetic energy dissipation, $\delta\epsilon$ (see Eq. 4), can contribute to both damping and driving of the pulsations (Gabriel 1996). The driving effect of $\delta\epsilon$ was shown by Dupret et al. (2005) for a γ Doradus star.

Summary

We discussed three different mode stability calculations of Delta Scuti stars which successfully reproduced the red edge of the IS. Each of these computations adopted a different time-dependent convection description. The results were discussed by comparing work integrals. All convection descriptions include, although in different ways, the perturbations of the turbulent fluxes. Gough (1977a), Xiong (1977, 1989), and Unno et al. (1989) did not include the contribution W_ϵ to the work integral because in the Boussinesq approximation (Spiegel & Veronis 1960) the viscous dissipation is neglected in the thermal energy equation. In practise, however, this term may be important. Grigahcène et al. (2005) included W_ϵ but ignored the damping contribution of the small-scale turbulence W_ν , which was found by Xiong & Deng (2001, 2007) to be the dominating damping term. The small-scale damping effect was also ignored in the calculations by Houdek (2000). A more detailed comparison of the convection descriptions has not yet been made but Houdek & Dupret have begun to address this problem.

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Discussion

Christensen-Dalsgaard: How does the mixing length affect the red edge of the γ Dor instability strip?

Houdek: The location of the red edge is predominantly determined by radiative damping which gradually dominates over the driving effect of the so-called convective flux blocking mechanism (Dupret et al. 2005). A change in the mixing length will not only affect the depth of the envelope convection zone but also the characteristic time scale of the convection and consequently the stability of g modes with different pulsation periods. A calibration of the mixing length to match the observed location of the γ Dor instability strip will also calibrate the depth of the convection zone at a given surface temperature.